

$$\begin{aligned}
 104 & \frac{5000}{R} \quad 1 \quad \frac{R}{100}^2 \quad 1 & [\text{From (1)}] \\
 104 & \frac{5000}{R} \frac{(1-R)^2 - (100)^2}{100000} \\
 104 & \frac{1}{R} (100-R-100)(100-R+100) \\
 104 & \frac{1}{2R} (200-R) \cdot R \\
 104 & \frac{200-R}{2} \\
 208 & \frac{200-R}{2} \quad R = 8\%
 \end{aligned}$$

### MCQ's

1. (b)  
5. (c)

2. (c)  
6. (d)

3. (b)  
7. (d)

4. (a)

## 7

## Algebraic Expressions and Factorisation

### Exercise 7.1

1. (a)  $2x - 7x \quad \{2, 7\} \quad \{x, x\} \quad 14 \quad x^2 - 14x^2$   
      (b)  $3x^2 - 6x^3 \quad \{3, 6\} \quad \{x^2, x^3\} \quad 18 \quad x^2 - 3 \quad 18x^5$   
      (c)  $(-7x^2) \cdot 2y \quad \{-7, 2\} \quad x^2 \cdot y \quad 14x^2y$   
      (d)  $\frac{3}{2}x^2y^2 - \frac{6}{7}xy^2 \quad \frac{3}{2}, \frac{6}{7} \quad \{x^2, x\} \quad \{y^2, y^2\}$   

$$\frac{18}{14} \quad \{x^2 - 1\} \quad \{y^2 - 2\}$$
  

$$\frac{18}{14}x^3y^4 - \frac{9}{7}x^3y^4$$
2. (a) Multiply  $3x$ ,  $4x^2$  and  $7x^3$   $3x \cdot 4x^2 \cdot 7x^3$   

$$\{3, 4, 7\} \quad \{x, x^2, x^3\}$$
  

$$84 \quad \{x^1, x^2, x^3\} \quad 84x^6$$
- (b) Multiply  $a^3$ ,  $6a^2b$  and  $2b^3$   $a^3 \cdot 6a^2b \cdot 2b^3$   

$$\{6, 2\} \quad \{a^3, a^2, b, b^3\}$$
  

$$12 \quad \{a^3, a^2\} \quad \{b^1, b^3\} \quad 12a^5b^4$$
- (c) Multiply  $16x^6$ ,  $10xy^2$  and  $\frac{3}{5}x^2y^2$   $16x^6 \cdot 10xy^2 \cdot \frac{3}{5}x^2y^2$   

$$16, 10, \frac{3}{5} \quad \{x^6, x, y^2, x^2, y^2\}$$
  

$$\{16, 2, 3\} \quad \{x^6, x, x^2\} \quad \{y^2, y^2\}$$
  

$$96 \{x^6, x^2\} \{y^2, y^2\}$$
  

$$96x^2y^4$$

(d) Multiply  $2p^4$ ,  $4p^4q^2$  and  $\frac{3}{8}pq^2$

$$2 \quad 4 \quad \frac{3}{8} \quad \{p^4 \quad p^4 \quad q^2 \quad p \quad q^2\}$$

$$\{3\} \quad \{p^4 \quad p^4 \quad p\} \quad \{q^2 \quad q^2\}$$

$$3 \quad \{p^4 \quad 4 \quad 1\} \quad \{q^2 \quad 2\}$$

$$3p^9q^4$$

3. (a)  $(2x - y)$  by  $(3x - 5)$

$$3x(2x - y) \quad 5y(2x - y)$$

$$6x^2 \quad 3y \quad 10xy \quad 5y^2$$

$$6x^2 \quad 13xy \quad 5y^2$$

(b) Multiply  $(3a - 2)$  by  $(2a - 5)$

$$2a(3a - 2) \quad 5(3a - 2)$$

$$6a^2 \quad 4a \quad 15a \quad 10$$

$$6a^2 \quad 11a \quad 10$$

(c) Multiply  $(p - q)$  by  $(p + q)$

$$p(p - q) \quad q(p + q)$$

$$p^2 \quad pq \quad pq \quad q^2$$

$$p^2 \quad q^2$$

(d) Multiply  $\frac{1}{2}x^2 - y^2$  by  $x^2 - \frac{1}{2}y^2$

$$x^2 \quad \frac{1}{2}x^2 \quad y^2 \quad \frac{1}{2}y^2 \quad \frac{1}{2}x^2 \quad y^2$$

$$\frac{1}{2}x^4 \quad x^2y^2 \quad \frac{1}{4}x^2y^2 \quad \frac{1}{2}y^4$$

$$\frac{1}{2}x^4 \quad \frac{3}{4}x^2y^2 \quad \frac{1}{2}y^4$$

4. (a) Product of  $(-3x)(2x^2 - 6x - 7)$

$$6x^3 \quad 18x^2 \quad 21x$$

(b) Product of  $\frac{1}{2}xy(x^2 - 2xy - y^2)$

$$\frac{1}{2}xy(x^2 - 2xy - y^2)$$

$$\frac{1}{2}x^3y \quad x^2y^2 \quad \frac{1}{2}xy^3$$

(c) Product of  $a^2(a^3 - 3a^2b - b^3 - 3ab^2)$

$$a^2(a^3 - 3a^2b - b^3 - 3ab^2)$$

$$a^5 \quad 3a^4b \quad a^2b^3 \quad 3a^3b^2$$

(d) Product of  $\frac{3}{5}p^2q(p^4 - q^4 - 2p^2q^2)$

$$\frac{3}{5}p^2q(p^4 - q^4 - wp^2q^2)$$

$$\frac{3}{5}p^6q \quad \frac{3}{5}p^2q^5 \quad \frac{6}{5}p^4q^3$$

5. (a) Multiply  $(a - b)$  by  $(a^2 - b^2)$

$$a(a^2 - b^2) \quad b(a^2 - b^2)$$

$$a^3 \quad ab^2 \quad ba^2 \quad b^3$$

(b) Multiply  $(p^2 - q^2)(p^2 + q^2)$

$$p^2(p^2 - q^2) \quad q^2(p^2 - q^2)$$

$$p^4 \quad p^2q^2 \quad p^2q^2 \quad q^4$$

$$p^4 \quad q^4$$

(c) Multiple  $\frac{3}{5}x - \frac{1}{3}y$  by  $\frac{3}{5}x - \frac{1}{3}y$

$$\begin{array}{r} \frac{3}{5}x \quad \frac{1}{3}y \\ \frac{3}{5}x \quad \frac{1}{3}y \\ \hline \frac{9}{25}x^2 \quad \frac{3}{15}xy \end{array}$$

$$\begin{array}{r} \frac{1}{3}y \quad \frac{3}{5}x \quad \frac{1}{3}y \\ \frac{3}{15}xy \quad \frac{3}{15}xy \quad \frac{1}{9}y^2 \\ \hline \frac{9}{25}x^2 \quad \frac{1}{9}y^2 \end{array}$$

(d) Multiple  $(p^2 - q^2)$  by  $(p^2 - pq - q^2)$

$$\begin{array}{r} (p^2 - q^2) \quad (p^2 - pq - q^2) \\ p^2(p^2 - pq - q^2) \quad q^2(p^2 - pq - q^2) \\ p^4 \quad p^3q \quad p^2q^2 \quad p^2q^2 \quad pq^3 \quad q^4 \\ p^4 \quad p^3q \quad 2p^2q^2 \quad pq^3 \quad q^4 \end{array}$$

6. (a) Multiply  $(a - 2b)$  by  $(2a - b)$

$$\begin{array}{r} a \quad 2b \\ 2a \quad b \\ \hline 4a^2 \quad 4ab \\ ab \quad 2b^2 \\ \hline 4a^2 \quad 5ab \quad 2b^2 \end{array}$$

(Multiply by  $2a$ )  
 (Multiply by  $b$ )  
 (Add the terms vertically)

(b) Multiply  $(p^2 - q^2)$  by  $(p^2 - q^2)$

$$\begin{array}{r} p^2 \quad q^2 \\ p^2 \quad q^2 \\ \hline p^4 \quad p^2q^2 \\ p^2q^2 \quad q^4 \\ \hline p^4 \quad q^4 \end{array}$$

(multiply by  $p^2$ )  
 (multiply by  $q^2$ )  
 (Add the terms vertically)

(c) Multiply  $\frac{3}{5}x - \frac{1}{3}y$  by  $\frac{3}{5}x - \frac{1}{3}y$

$$\begin{array}{r} \frac{3}{5}x \quad \frac{1}{3}y \\ \frac{3}{5}x \quad \frac{1}{3}y \\ \hline \frac{9}{25}x^2 \quad \frac{3}{15}yx \\ \frac{3}{15}yx \quad \frac{1}{9}y^2 \\ \hline \frac{9}{25}x^2 \quad \frac{1}{9}y^2 \end{array}$$

(multiply by  $\frac{3}{5}x$ )  
 (multiply by  $\frac{1}{3}y$ )  
 (Add the terms vertically)

(d)  $(x - y)(x^2 - y^2 - xy)$

$$\begin{array}{r} x^2 \quad y^2 \quad yx \\ x \quad y \\ \hline x^3 \quad xy^2 \quad yx^2 \\ y^2x \quad yx^2 \quad y^3 \\ \hline x^3 \quad y^3 \end{array}$$

(Multiply by  $x$ )  
 (Add the terms vertically)

7. (a)  $c(b-a) b(a-c) a(b-c) bc ac ba bc ab ac 0$   
      (b)  $x(x-y^2-z) y^2(x-y-z) z(x-y^2)$   
 $\quad\quad\quad x^2 \quad xy^2 \quad zx \quad y^2x \quad y^3 \quad zy^2 \quad zx \quad zy^2$   
 $\quad\quad\quad x^2 \quad xy^2 \quad y^2x \quad y^3 \quad yz \quad zy^2$
8. (a)  $6x^2y(z^2-y^2) \quad 6x^2y \quad (z^2-y^2)$   
 $(x-1, y-2 \text{ and } z-3)$   
 $\quad\quad\quad 6(1)^2(2)(3)^2 \quad 6(1)^2(2)^3$   
 $\quad\quad\quad 6 \quad 1 \quad 2 \quad 9 \quad 6 \quad 1 \quad 8$   
 $\quad\quad\quad 108 \quad 48 \quad 60$
- (b)  $(4y-z)(z-4y) \quad (4y-z) \quad (z-4y)$   
 $\quad\quad\quad 4y(z-4y) \quad z(z-4y)$   
 $\quad\quad\quad 4yz \quad 16y^2 \quad z^2 \quad 4zy$   
 $\quad\quad\quad z^2 \quad 16y^2$
- $(z-3 \text{ and } y-2)$   
 $\quad\quad\quad (3)^2 \quad 16(2)^2$   
 $\quad\quad\quad 9 \quad 16 \quad 4 \quad 9 \quad 64 \quad 55$
- (c)  $(2x-2y)^2 \quad 4x^2 \quad 4y^2 \quad 8xy$   
 $(x-1 \text{ and } y-2)$   
 $\quad\quad\quad 4(1)^2 \quad 4(2)^2 \quad 8 \quad 1 \quad 2$   
 $\quad\quad\quad 4 \quad 4 \quad 4 \quad 16$   
 $\quad\quad\quad 4 \quad 16 \quad 16 \quad 20 \quad 16 \quad 4$
- (d)  $(x-y-z)(x-y-z) \quad x(x-y-z) \quad y(x-y-z) \quad z(x-y-z)$   
 $\quad\quad\quad x^2 \quad yx \quad zx \quad yx \quad y^2 \quad zy \quad zx \quad yz \quad z^2$   
 $\quad\quad\quad x^2 \quad y^2 \quad z^2 \quad 2xy \quad 2zx \quad 2yz \quad (x-1, y-2 \text{ and } z-2)$   
 $\quad\quad\quad (1)^2 \quad (2)^2 \quad (3)^2 \quad 2 \quad 1 \quad 2 \quad 2 \quad 3 \quad 1 \quad 2 \quad 2 \quad 3$   
 $\quad\quad\quad 1 \quad 4 \quad 9 \quad 4 \quad 6 \quad 12 \quad 36$
9. Product of  $(2x-3y)$  and  $(x^2-2xy-y^2)$   
 $(2x-3y) \quad (x^2-2xy-y^2)$   
 $2x(x^2-2xy-y^2) \quad 3y(x^2-2xy-y^2)$   
 $2x^3 \quad 4x^2y \quad 2xy^2 \quad 3x^2y \quad 6xy^2 \quad 3y^3$   
 $2x^3 \quad 7x^2y \quad 8xy^2 \quad 3y^3$
- Verification :**  $(2x-3y)(x^2-2xy-y^2) = 2x^3 - 7x^2y - 8xy^2 - 3y^3$
- LHS  $(2x-3y)(x^2-2xy-y^2)$   
 $(2-1 \quad 3-2)[(1)^2 \quad 2-1 \quad 2-(2)^2][x-(1), y-z]$   
 $(-2 \quad 6)(1-4 \quad 4)$   
 $4(1 \quad 4 \quad 4)$   
 $4 \quad 1 \quad 4$
- RHS  $2x^3 - 7x^2y - 8xy^2 - 3y^3$   
 $2(-1)^3 \quad 7(-1)^2 \quad 2 \quad 8 \quad 1 \quad (2)^2 \quad 3(2)^3$   
 $2 \quad 7 \quad 1 \quad 2 \quad 8 \quad 4 \quad 3 \quad 8$   
 $2 \quad 14 \quad 32 \quad 24 \quad 38 \quad 34 \quad 4$

Hence, LHS = RHS verified.

10. Product of  $(x^3 - 2x^2 - 5x - 1)$  and  $(x^2 - 7x - 1)$

$$\begin{aligned}
 & (x^3 - 2x^2 - 5x - 1) \quad (x^2 - 7x - 1) \\
 & x^2(x^3 - 2x^2 - 5x - 1) - 7x(x^3 - 2x^2 - 5x - 1) - 1(x^3 - 2x^2 - 5x - 1) \\
 & (x^4 - 2x^4 - 5x^3 - x^2) - (7x^4 - 14x^3 - 35x^2 - 7x) - (x^3 - 2x^2 - 5x - 1) \\
 & x^5 - 2x^4 - 7x^4 - 5x^3 - 14x^3 - x^3 - x^2 - 35x^2 - 2x^2 - 7x - 5x - 1 \\
 & x^5 - 9x^4 - 10x^3 - 32x^2 - 2x - 1
 \end{aligned}$$

### Exercise 7.2

1. (a)  $3\sqrt{y} - 4y - 7y^2 - 3(y)^{1/2} - 4y - 7y^2$  It is not a polynomial.  
 (b)  $\sqrt{2x} - x^2 - x^3$  It is a polynomial in variable  $x$ .  
 (c)  $\frac{-x^2}{3} - 4x - 12$  It is a polynomial in variable  $x$ .

(d)  $2x^{-2} - 3x^{-1} - 5 - 4x - \frac{2-1}{x^2} - \frac{3-1}{x} - 5 - 4x$  It is not a polynomial.

(e)  $\sqrt{ax^{1/2}} - ax - 7x^2 - 5$  It is not a polynomial.

(f)  $x^3 - x^3 - x^3 - \frac{1}{x^3}$  It is not a polynomial.

2. (a) Divide  $12x^2 y^3$  by  $3xy$

$$12x^2 y^3 \quad 3xy \quad \frac{12x^2 y^3}{3xy} = 4xy^2$$

- (b) Divide  $36abc^2$  by  $(-9ac)$

$$36abc^2 \quad 9ac \quad \frac{36abc^2}{9ac} = 4bc$$

- (c) Divide  $60p^2 q^2 r^2$  by  $12pq r^2$

$$60p^2 q^2 r^2 \quad 12pq r^2 \quad \frac{60p^2 q^2 r^2}{12pq r^2} = 5pq$$

- (d) Divide  $25x^2 yz$  by  $3xyz$

$$25x^2 yz \quad 3xyz \quad \frac{25x^2 yz}{3xyz} = \frac{25}{3}x$$

3. (a)  $5x^3 - 30x^2 - 45x$  divide by  $5x$

$$\begin{array}{r}
 5x^3 - 30x^2 - 45x \quad 5x \\
 \hline
 5x \quad \quad \quad 5x \\
 \overline{5x^3} \quad \overline{30x^2} \quad \overline{45x} \\
 \overline{5x} \quad \overline{5x} \quad \overline{5x} \\
 \overline{x^2} \quad \overline{6x} \quad \overline{9}
 \end{array}$$

- (b)  $8x^2 y^2 - 6xy^2 - 10x^2 y^3$  divide by  $2xy$

$$\begin{array}{r}
 8x^2 y^2 - 6xy^2 - 10x^2 y^3 \quad 2xy \\
 \hline
 2xy \quad \quad \quad 2xy \quad \quad \quad 2xy \\
 \overline{8x^2 y^2} \quad \overline{6xy^2} \quad \overline{10x^2 y^3} \\
 \overline{2xy} \quad \overline{2xy} \quad \overline{2xy} \\
 4xy - 3y - 5xy^2
 \end{array}$$

(c)  $4x^3 - 8x^2 - 2$  divide by  $(-2x)$

$$\begin{array}{r} 4x^3 - 8x^2 - 2 \\ \hline -2x \end{array}$$

$$\begin{array}{r} 4x^3 - 8x^2 - 2 \\ \hline -2x \end{array}$$

$$\begin{array}{r} 4x^3 - 8x^2 - 2 \\ \hline -2x \end{array}$$

$$2x^2 - 4x - x$$

(d)  $10a^2b - 6ab - 12ab^2$  by  $3ab$

$$\begin{array}{r} 10a^2b - 6ab - 12ab^2 \\ \hline 3ab \end{array}$$

$$\begin{array}{r} 10a^2b - 6ab - 12ab^2 \\ \hline 3ab \end{array}$$

$$\begin{array}{r} 10a^2b - 6ab - 12ab^2 \\ \hline 3ab \end{array}$$

$$\frac{10}{3}a^2 - 2 - 4b$$

4. (a)  $(x^2 - 12x - 35)$  divide by  $(x - 5)$

$$\begin{array}{r} x - 7 \end{array} \overline{\smash[b]{x^2 - 12x - 35}} \begin{array}{l} x - 5 \\ x^2 - 7x \\ \hline 5x - 35 \\ 5x - 35 \\ \hline 0 \end{array}$$

(b)  $6x^2 - 13x - 6$  divide by  $(2x - 3)$

$$\begin{array}{r} 2x - 3 \end{array} \overline{\smash[b]{6x^2 - 13x - 6}} \begin{array}{l} 3x - 2 \\ 6x^2 - 9x \\ \hline 4x - 6 \\ 4x - 6 \\ \hline 0 \end{array}$$

(c)  $12x^3 - 20x^2 - 9x - 15$  divide by  $(3x - 5)$

$$\begin{array}{r} 3x - 5 \end{array} \overline{\smash[b]{12x^3 - 20x^2 - 9x - 15}} \begin{array}{l} 4x^2 - 3 \\ 12x^2 - 20x^2 \\ \hline 0 \end{array}$$

$$\begin{array}{r} 3x - 5 \end{array} \overline{\smash[b]{12x^3 - 20x^2 - 9x - 15}} \begin{array}{l} 4x^2 - 3 \\ 12x^2 - 20x^2 \\ \hline 0 \end{array}$$

$$\begin{array}{r} 3x - 5 \end{array} \overline{\smash[b]{12x^3 - 20x^2 - 9x - 15}} \begin{array}{l} 4x^2 - 3 \\ 12x^2 - 20x^2 \\ \hline 0 \end{array}$$

(d)  $a^3 - 6a^2 - 11a - 6$  divide by  $(a^2 - 5a - 6)$

$$\begin{array}{r} a^2 - 5a - 6 \end{array} \overline{\smash[b]{a^3 - 6a^2 - 11a - 6}} \begin{array}{l} a - 1 \\ a^3 - 5a^2 - 6a \\ \hline a^2 - 5a - 6 \\ a^2 - 5a - 6 \\ \hline 0 \end{array}$$

$$\begin{array}{r} a^2 - 5a - 6 \end{array} \overline{\smash[b]{a^3 - 6a^2 - 11a - 6}} \begin{array}{l} a - 1 \\ a^3 - 5a^2 - 6a \\ \hline a^2 - 5a - 6 \\ a^2 - 5a - 6 \\ \hline 0 \end{array}$$

$$\begin{array}{r} a^2 - 5a - 6 \end{array} \overline{\smash[b]{a^3 - 6a^2 - 11a - 6}} \begin{array}{l} a - 1 \\ a^3 - 5a^2 - 6a \\ \hline a^2 - 5a - 6 \\ a^2 - 5a - 6 \\ \hline 0 \end{array}$$

(e)  $p^4 - p^2 - 1$  divide by  $p^2 - p - 1$

$$\begin{array}{r}
 p^2 - p - 1) \overline{p^4 - p^2 - 1} \\
 \underline{( ) p^4 ( ) - p^3 ( ) + p^2} \\
 \qquad\qquad\qquad p^3 - 1 \\
 \underline{( ) - p^3 ( ) + p^2} \\
 \qquad\qquad\qquad p^2 - p - 1 \\
 \underline{p^2 - p - 1} \\
 \qquad\qquad\qquad ( ) - ( ) ( ) \\
 \hline
 0
 \end{array}$$

(f)  $\frac{6y^5 - 4y^4 - 3y^3 - 1}{3y^2 - y}$  divide  $(3^2 - y - 1)$

$$\begin{array}{r}
 6y^5 - 4y^4 - 3y^3 - 1 \\
 \underline{6y^5 - 4y^4 - 3y^3 - 1} \\
 \qquad\qquad\qquad 0 \\
 \underline{( ) - ( ) - ( )} \\
 \qquad\qquad\qquad 6y^4 - 5y^3 - 1 \\
 \qquad\qquad\qquad 6y^4 - 2y^3 - 2y^2 \\
 \underline{( ) - ( ) - ( )} \\
 \qquad\qquad\qquad -3y^3 - 2y^2 - 1 \\
 \qquad\qquad\qquad 3y^3 - y^2 - y - 1 \\
 \underline{( ) - ( ) - ( ) - ( )} \\
 \qquad\qquad\qquad 3y^2 - y - 1 \\
 \qquad\qquad\qquad 3y^2 - y - 1 \\
 \underline{( ) - ( ) - ( )} \\
 \qquad\qquad\qquad 0
 \end{array}$$

5. (a) Divide  $4x^3 - x - 1$  by  $(2x - 1)$

$$\begin{array}{r}
 2x - 1) \overline{4x^3 - x - 1} \\
 \underline{4x^3 - 2x^2} \\
 \qquad\qquad\qquad 2x^2 - x \\
 \underline{( ) - ( )} \\
 \qquad\qquad\qquad 2x^2 - x \\
 \underline{2x^2 - x} \\
 \qquad\qquad\qquad ( ) - ( ) \\
 \hline
 1
 \end{array}$$

(b) Divide  $2a^3 - 5a^2 - 8a - 4$  by  $(2a - 1)$

$$\begin{array}{r}
 2a - 1) \overline{2a^3 - 5a^2 - 8a - 4} \\
 \underline{2a^3 - a^2} \\
 \qquad\qquad\qquad 4a^2 - 8a - 4 \\
 \underline{( ) - ( )} \\
 \qquad\qquad\qquad 4a^2 - 2a \\
 \underline{( ) - ( )} \\
 \qquad\qquad\qquad 6a - 4 \\
 \underline{6a - 3} \\
 \qquad\qquad\qquad ( ) - ( ) \\
 \hline
 1
 \end{array}$$

6. (a) Divide  $m^4 - m^3 - m^2$  by  $m - 1$

$$\begin{array}{r} m-1 \overline{)m^4 - m^3 - m^2} \\ m^4 - m^3 \\ \hline m^2 \end{array}$$

(b) Divide  $(x^4 - 1)$  by  $(x - 1)$

$$\begin{array}{r} x-1 \overline{x^4 - 1} \\ x^4 - x^3 \\ \hline ( ) - ( ) \\ x^3 - 1 \\ x^3 - x^2 \\ \hline ( ) - ( ) \\ x^2 - 1 \\ x^2 - x \\ \hline ( ) - ( ) \\ x - 1 \\ x - 1 \\ \hline ( ) - ( ) \\ 2 \end{array}$$

Quotient  $x^3 - x^2 - x - 1$ ; Remainder 2

Verification

Dividend      (Divisor  $\times$  quotient)      remainder

$$(x - 1)(x^3 - x^2 - x - 1) + 2$$

$$x(x^3 - x^2 - x - 1) + 1(x^3 - x^2 - x - 1) + 2$$

$$x^4 - x^3 - x^2 - x + x^3 - x^2 - x - 1 + 2$$

$$x^4 - 1 + 2$$

$$x^4 - 1$$

7.  $(4x^4 - 2x^3 - 6x^2 - x - 5) \div 2x^2 - x - 2$

$$\begin{array}{r} 2x^2 - x - 2 \overline{)4x^4 - 2x^3 - 6x^2 - x - 5} \\ 4x^2 - 2x^3 - 4x^2 \\ \hline ( ) - ( ) - ( ) \\ 4x^3 - 2x^2 - x - 5 \\ 4x^3 - 2x^2 - 4x \\ \hline ( ) - ( ) - ( ) \\ 3x - 5 \end{array}$$

8.  $x^4 - x^3 - 8x^2 - ax - b$  dividend by  $x^2 - 1$

$$\begin{array}{r} x^2 - 1 \overline{)4x^4 - 2x^3 - 6x^2 - x - 5} \\ x^4 - x^2 \\ \hline x^3 - 7x^2 - ax - b \\ x^3 - 7x^2 - 1x - 7 \\ \hline ( ) - ( ) - ( ) - ( ) \end{array}$$

For the remainder to be zero than  $a = 1, b = 7$

Hence,  $a$ 's values 1,  $b$ 's value 7

9.  $t^3 - 2t^2 - 3t - 18$  divide by  $t - 3$

$$\begin{array}{r} t - 3 \) t^3 - 2t^2 - 3t - 18 \\ \underline{-} t^2 - 3t^2 \\ \underline{( ) \quad ( )} \\ t^2 - 3t - 18 \\ \underline{-} t^2 - 3t \\ \underline{( )} \\ 6t - 18 \\ \underline{-} 6t - 18 \\ \underline{( ) \quad ( )} \\ 0 \end{array}$$

Yel, the quotient  $t^2 - t - 6$  Remainder 0

### Exercise 7.3

1. (a)  $(5x - 3y)^2$

$$\because (a - b)^2 = a^2 - b^2 - 2ab$$

Here,  $a = 5x$  and  $b = 3y$

$$(5x - 3y)^2 = (5x)^2 - (3y)^2 - 2(5x)(3y)$$
$$= 25x^2 - 9y^2 - 30xy.$$

(b)  $(5 - 12x^2)^2$

$$\because (a - b)^2 = a^2 - b^2 - 2ab$$

Here  $a = 5$  and  $b = 12x^2$

$$(5 - 12x^2)^2 = (5)^2 - (12x^2)^2 - 2(5)(12x^2)$$
$$= 25 - 144x^4 - 120x^2.$$

(c)  $5x - \frac{1}{5y}^2$

$$\because (a - b)^2 = a^2 - b^2 - 2ab$$

Here  $a = 5x$ ,  $b = \frac{1}{5y}$

$$5x - \frac{1}{5y}^2 = (5x)^2 - \frac{1}{5y}^2 - 2(5x) \cdot \frac{1}{5y}$$
$$= 25x^2 - \frac{1}{25y^2} - \frac{2x}{y}$$

(d)  $(3x - 4y)^2$

Here,  $a = 3x$ ,  $b = 4y$

$$\because (a - b)^2 = a^2 - b^2 - 2ab$$

$$(3x - 4y)^2 = (3x)^2 - (4y)^2 - 2(3x)(4y)$$
$$= 9x^2 - 16y^2 - 24xy.$$

$$(e) \quad \sqrt{3}x - \frac{1}{5}y^2$$

Here  $a = \sqrt{3}x$ ,  $b = \frac{1}{5}y$

$$\therefore (a - b)^2 = a^2 + b^2 - 2ab$$

$$\begin{aligned} \sqrt{3}x - \frac{1}{5}y^2 &= (\sqrt{3}x)^2 + \left(\frac{1}{5}y\right)^2 - 2(\sqrt{3}x)\left(\frac{1}{5}y\right) \\ &= 3x^2 + \frac{1}{25}y^2 - \frac{2\sqrt{3}}{5}xy. \end{aligned}$$

$$(f) \quad (x - 3y)(x - 3y)$$

$$\begin{aligned} (x - 3y)(x - 3y) &= (x - 3y)^2 \\ &= x^2 - (3y)^2 - 2(x)(3y) \\ &= x^2 - 9y^2 - 6xy. \end{aligned}$$

$$2. \quad (a) \quad 9x^2 - 49y^2 - 42xy$$

Here  $x = 3$  and  $y = 1$

$$\begin{aligned} 9x^2 - 49y^2 - 42xy &= (3x)^2 - (7y)^2 - 2(3x)(7y) \\ &= (3x - 7y)^2 \\ &= (3 - 7)^2 \quad (\text{when } x = 3, y = 1) \\ &= (9 - 49)^2 = 256. \end{aligned}$$

$$(b) \quad 25x^2 - 64y^2 - 80xy$$

Here,  $x = 4$ ,  $y = 2$

$$\begin{aligned} 25x^2 - 64y^2 - 80xy &= (5x)^2 - (8y)^2 - (2)(5x)(8y) \\ &= (5x - 8y)^2 \\ &= (5 - 8)^2 \quad (\because x = 4 \text{ and } y = 2) \\ &= (20 - 16)^2 = (4)^2 = 16 \end{aligned}$$

$$3. \quad (a) \quad 2x - 3y - 8$$

On squaring both sides

$$\begin{aligned} (2x - 3y)^2 - (8)^2 &= 0 \\ 4x^2 - 9y^2 - 2(2x)(3y) - 64 &= 0 \\ 4x^2 - 9y^2 - 12(xy) - 64 &= 0 \\ 4x^2 - 9y^2 - 12(2) - 64 &= 0 \quad (\because xy = 2) \\ 4x^2 - 9y^2 - 64 - 24 &= 0 \\ 4x^2 - 9y^2 - 40 &= 0. \end{aligned}$$

$$(b) \quad 3x - 7y - 8 \text{ on squaring both sides}$$

$$\begin{aligned} (3x - 7y)^2 - (8)^2 &= 0 \\ 9x^2 - 49y^2 - 2(3x)(7y) - 64 &= 0 \\ 9x^2 - 49y^2 - 42(xy) - 64 &= 0 \\ 9x^2 - 49y^2 - 42(-1) - 64 &= 0 \\ 9x^2 - 49y^2 - 42 - 64 &= 0 \end{aligned}$$

$$\begin{array}{r} 9x^2 \quad 49y^2 \quad 64 \quad 42 \\ 9x^2 \quad 49y^2 \quad 22 \end{array}$$

4. (i)  $x - \frac{1}{x} = 6$

Squaring both sides

$$\begin{aligned} x - \frac{1}{x} &= 6^2 \\ x^2 - \frac{1}{x^2} - 2 &= x - \frac{1}{x} + 36 \\ x^2 - \frac{1}{x^2} - 2 &= 36 \\ x^2 - \frac{1}{x^2} &= 36 + 2 \\ x^2 - \frac{1}{x^2} &= 34. \end{aligned}$$

(ii)  $x^2 - \frac{1}{x^2} = 34$  squaring on both side

$$\begin{aligned} x^2 - \frac{1}{x^2} &= 34^2 \\ (x^2)^2 - \frac{1}{x^2} &= 2 \quad x^2 - \frac{1}{x^2} = 1156 \\ x^4 - \frac{1}{x^4} &= 2 \quad 1156 \\ x^4 - \frac{1}{x^4} &= 1156 - 2 = 1154. \end{aligned}$$

5. (a)  $x - \frac{1}{x} = 5$  squaring on both sides

$$\begin{aligned} x - \frac{1}{x} &= 5^2 \\ x^2 - \frac{1}{x^2} - 2 &= 25 \\ x^2 - \frac{1}{x^2} &= 25 + 2 \\ x^2 - \frac{1}{x^2} &= 27. \end{aligned}$$

(b)  $x^2 - \frac{1}{x^2} = 27$  squaring on both sides

$$\begin{aligned} x^2 - \frac{1}{x^2} &= 27^2 \\ (x^2)^2 - \frac{1}{x^2} &= 2 \quad x^2 - \frac{1}{x^2} = 729 \end{aligned}$$

$$\begin{array}{r}
 x^4 \quad \frac{1}{x^4} \quad 2 \quad 729 \\
 x^4 \quad \frac{1}{x^4} \quad 729 \quad 2 \\
 x^4 \quad \frac{1}{x^4} \quad 727.
 \end{array}$$

6. (a)  $(4x - 5y)(4x - 5y) = (4x)^2 - (5y)^2 = 16x^2 - 25y^2$  [ $\because (a - b)(a - b) = a^2 - b^2$ ]
- (b)  $(ab - cd)(ab - cd) = (ab)^2 - (cd)^2$  [ $\because (a - b)(a - b) = a^2 - b^2$ ]
- (c)  $(ab - cd)(ab - cd) = (ab)^2 - (cd)^2 = (x^4 - 1)(x^4 - 1)$   
 $= (x^4)^2 - (1)^2 = x^8 - 1$  [ $\because (a - b)(a - b) = a^2 - b^2$ ]
- (d)  $x - \frac{y}{5} = 1 \quad x + \frac{y}{5} = 1 \quad x - \frac{y}{5} = (1)^2 = 1$  [ $\because (a - b)(a - b) = a^2 - b^2$ ]  
 $x^2 - \frac{y^2}{25} = \frac{2xy}{5} = 1$  [ $\because (a - b)^2 = a^2 - b^2 - 2ab$ ]

7. (a)  $(103)^2 = (100 + 3)^2$   
We know that  $(a + b)^2 = a^2 + b^2 + 2ab$   
then  $(100 + 3)^2 = (100)^2 + (3)^2 + 2(100)(3)$   
 $= 10000 + 9 + 600$   
 $= 106009.$

- (b)  $(91)^2 = (100 - 9)^2$   
We know that,  $(a - b)^2 = a^2 + b^2 - 2ab$   
then,  $(100 - 9)^2 = (100)^2 + (9)^2 - 2(100) - (9)$   
 $= 10000 - 81 - 1800 - 10081 + 1800 = 8281.$
- (c)  $(0.98)^2 = (1 - 0.02)^2$   
then  $(1 - 0.02)^2 = (1)^2 + (0.02)^2 - 2(1) - (0.02)$   
 $= 1 - 0.0004 - 0.04 = 0.9604.$

- (d)  $(97)^2 = (100 - 3)^2$   
We know that,  $(a - b)^2 = a^2 + b^2 - 2ab$   
 $(100 - 3)^2 = (100)^2 + (3)^2 - 2 \cdot 100 \cdot 3$   
 $= 10000 + 9 + 600$   
 $= 10009 + 600 = 9409.$

- (e)  $103 \cdot 97$   
We know that,  $a^2 - b^2 = (a + b)(a - b)$   
 $103 \cdot 97 = (100 + 3)(100 - 3) = (100)^2 - (3)^2$   
 $= 10000 - 9 = 9991$

- (f)  $104 \cdot 104 = (100 + 4)^2$   
 $= (100)^2 + (4)^2 + 2 \cdot 100 \cdot 4$   
 $= 10000 + 16 + 800 = 10816.$

$$(g) \quad 166 \quad 166 \quad 134 \quad 134 \quad (166)^2 \quad (134)^2 \\ (166 \quad 134)(166 \quad 134) \quad [:(a^2 - b^2)(a - b)] \\ (300)(32) \quad 9600.$$

$$(h) \quad 0.78 \quad 0.78 \quad 0.22 \quad 0.22 \\ (0.78)^2 \quad (0.22)^2 \\ (0.78 \quad 0.22)(0.78 \quad 0.22) \quad [:(a^2 - b^2)(a - b)] \\ (1)(0.56) = 0.56.$$

$$(i) \quad 0.54 \quad 0.54 \quad 0.46 \quad 0.46 \quad (0.54)^2 \quad (0.46)^2 \\ (0.54 \quad 0.46)(0.54 \quad 0.46) \quad [:(a^2 - b^2)(a - b)] \\ (1.00)(0.08) \quad 0.08$$

### Exercise 7.4

1. (a)  $(x - 2y - 5z)^2$   
 $\because (a - b - c)^2 = a^2 - b^2 - c^2 + 2ab - 2bc - 2ac$   
 $(x - 2y - 5z)^2 = (x)^2 - (2y)^2 - (5z)^2 - 2(x - 2y)$   
 $x^2 - 4y^2 - 25z^2 - 4xy - 20yz - 10xz$

(b)  $\frac{1}{4}x - \frac{1}{2}y - 16^2$   
 $\because (a - b - c)^2 = a^2 - b^2 - c^2 + 2ab - 2bc - 2ca$   
 $\frac{1}{4}x - \frac{1}{2}y - 16^2 = \frac{1}{4}x^2 - \frac{1}{2}y^2 - (16)^2 - 2 \cdot \frac{1}{4}x \cdot -\frac{1}{2}y$   
 $\frac{1}{16}x^2 - \frac{1}{4}y^2 - 256 - \frac{1}{4}xy - 16y - 8x.$

(c)  $\frac{a}{b} - \frac{b}{c} - \frac{c}{d}^2$   
 $\because (a - b - c)^2 = a^2 - b^2 - c^2 + 2ab - 2bc - 2ca$   
then  $\frac{a}{b} - \frac{b}{c} - \frac{c}{d}^2 = \frac{a}{b}^2 - \frac{b}{c}^2 - \frac{c}{d}^2 - 2 \cdot \frac{a}{b} \cdot -\frac{b}{c}$   
 $2 \cdot \frac{b}{c} \cdot -\frac{c}{d} - 2 \cdot \frac{c}{d} \cdot \frac{a}{b}$   
 $\frac{a^2}{b^2} - \frac{b^2}{c^2} - \frac{c^2}{d^2} - 2 \cdot \frac{a}{c} - 2 \cdot \frac{b}{d} - 2 \cdot \frac{ac}{bd}.$

2.  $x^2 - 4y^2 - 9z^2 - 4xy - 12yz - 6xz$   
 $x^2 - 4y^2 - 9z^2 - 4xy - 12yz - 6xz$   
 $(x)^2 - (2y)^2 - (3z)^2 - 2(x - 2y) - 2(2y - 3z) - 2(3z - x)$   
 $(x - 2y - 3z)^2 \quad [:(a - b - c)^2 = a^2 - b^2 - c^2 - 2ab - 2bc - 2ca]$

Now, on putting the value of  $x, y$  and  $z$ .

$x = 8, y = 7 \text{ and } z = 6$



$$\text{then } (x - 2y - 3z)^2 = [8(2) - 7(3) - 6(6)]^2 = (8 - 14 - 18)^2 = (40)^2 = 1600.$$

$$3. \quad x^2 - 4y^2 - 25z^2 - 4xy - 20yz - 10xz \\ (x^2 - (2y)^2 - (5z)^2 - 2(x - 2y) - 2(-2y - 5z) - 2(x - 5z)) \\ \{(x - (2y) - (5z))^2 - (x - 2y - 5z)^2 \\ [::(a - b - c)^2 = a^2 - b^2 - c^2 - 2ab + 12b - 2ca]$$

Now, on putting the value of  $x, y$  and  $x = 9, y = 2$  and  $z = 1$

$$\text{then } (x - 2y - 5z)^2 = \{9 - (2 - 2) - (5 - 1)\}^2 = [9 - 4 - 5]^2 = [9 - 9]^2 = [0]^2 = 0.$$

$$4. \quad x + y + z = 12 \text{ squaring both the sides.}$$

$$(x + y + z)^2 = (12)^2 \\ x^2 + y^2 + z^2 - 2xy - 2yz - 2zx = 144 \\ (x^2 + y^2 + z^2) - (2xy + 2yz + 2zx) = 144 \\ \therefore x^2 + y^2 + z^2 = 64 \\ 64 - (2xy + 2yz + 2zx) = 144 \\ 2xy + 2yz + 2zx = 144 - 64 = 80 \\ 2(xy + yz + zx) = 80 \\ xy + yz + zx = \frac{80}{2} = 40.$$

$$5. \quad x + y + z = 8 \text{ squaring both sides.}$$

$$(x + y + z)^2 = (8)^2 \\ x^2 + y^2 + z^2 - 2xy - 2yz - 2zx = 64 \\ x^2 + y^2 + z^2 - 2(xy + yz + zx) = 64 \\ x^2 + y^2 + z^2 - 2(13) = 64 \\ x^2 + y^2 + z^2 = 26 = 64 \\ x^2 + y^2 + z^2 = 64 - 26 = 38. \quad (\because xy + yz + zx = 13)$$

$$6. \quad x^2 + y^2 + z^2 = 35 \text{ on using identity}$$

$$(x + y + z)^2 = x^2 + y^2 + z^2 - 2xy - 2yz - 2zx \\ (x + y + z)^2 = (x^2 + y^2 + z^2) - 2(xy + yz + zx) \\ (x + y + z) = \sqrt{35 - 2(23)} \quad (\because x^2 + y^2 + z^2 = 35 \text{ and } xy + yz + zx = 23) \\ \sqrt{35 - 46} = \sqrt{81} \quad (x + y + z = 9)$$

$$7. \quad (a) \quad (2x + p + c)^2 = (2x + p + c)^2$$

$$(2x + p + c - 2x - p - c). (2x + p + c - 2x - p - c) \\ [::(a^2 - b^2) = (a - b)(a + b)]$$

$$\begin{aligned} & 4x(2p + 2c) \\ & 4x(2(p + c)) \end{aligned}$$

$$(b) \quad (x^2 + y^2 + z^2)^2 = (x^2 + y^2 + z^2)^2 \\ (x^2 + y^2 + z^2 - x^2 - y^2 - z^2). (x^2 + y^2 + z^2 - x^2 - y^2 - z^2)$$

$$[\because a^2 - b^2 = (a+b)(a-b)]$$

$$\begin{matrix} 2x & (2y^2 & 2z^2) \\ 4x^2 & (y^2 & z^2) \\ 4x^2 & (y & z)(y & z) \end{matrix}$$

$$\begin{aligned} (c) \quad & (a-b-c)^2 = (a-b-c)^2 = (a-b-c)^2 \\ \therefore \quad & (a-b-c)^2 = a^2 - b^2 - 2ab - 2bc + 2ca \\ & (a-b-c)^2 = (a-b-c)^2 = (a-b-c)^2 \\ & (a^2 - b^2 - 2ab - 2bc + 2ca) = (a^2 - b^2 - c^2 - 2ab - 2bc + 2ca) \\ & (a^2 - b^2 - c^2 - 2ab - 2bc + 2ca) = (a^2 - b^2 - c^2 - 2ab - 2bc + 2ca) \\ & a^2 - b^2 - c^2 - 2ab - 2bc + 2ca = a^2 - b^2 - c^2 - 2ab - 2bc + 2ca \\ & 3a^2 - 3b^2 - 3c^2 - 2ab - 2ca - 2bc \\ & 3(a^2 - b^2 - c^2) - 2ab - 2bc - 2ca. \end{aligned}$$

### Exercise 7.5

1. (a)  $(3x - 2y)^3$   
 $\therefore (a-b)^3 = a^3 - b^3 - 3ab(a-b)$   
 $(3x - 2y)^3 = 27x^3 - 8y^3 - 3(3x - 2y)(3x - 2y)$   
 $27x^3 - 8y^3 - (18xy)(3x - 2y) = 27x^3 - 8y^3 - 54x^2y + 36xy^2.$
- (b)  $\frac{1}{3}x - \frac{5}{3}y^3$   
 $\therefore (a-b)^3 = a^3 - b^3 - 3ab(a-b)$   
 $\frac{1}{3}x - \frac{5}{3}y^3 = \frac{1}{27}x^3 - \frac{125}{27}y^3 - 3 \cdot \frac{1}{3}x \cdot \frac{5}{3}y = \frac{1}{3}x - \frac{5}{3}y$   
 $\frac{1}{27}x^3 - \frac{125}{27}y^3 - \frac{5}{3}xy \cdot \frac{1}{3}x - \frac{5}{3}y$   
 $\frac{1}{27}x^3 - \frac{125}{27}y^3 - \frac{5}{9}x^2y - \frac{25}{9}xy^2.$
- (c)  $\frac{1}{3x} - \frac{2}{5y}^3$   
 $\therefore (a-b)^2 = a^3 - b^3 - 3ab(a-b)$   
 $\frac{1}{3x} - \frac{2}{5y}^3 = \frac{1}{27x^3} - \frac{8}{125y^3} - 3 \cdot \frac{1}{3x} \cdot \frac{2}{5y} = \frac{1}{3x} - \frac{2}{5y}$   
 $\frac{1}{27x^3} - \frac{8}{125y^3} - \frac{2}{5xy} \cdot \frac{1}{3x} - \frac{2}{5y}$   
 $\frac{1}{27x^3} - \frac{8}{125y^3} - \frac{2}{15x^2y} - \frac{4}{25xy^2}.$
2. (a)  $(1004)^3 - (1000 - 4)^3$   
 $\therefore (a-b)^3 = a^3 - b^3 - 3ab(a-b)$

$$(1000 - 4)^3 = (1000)^3 - (4)^3 + 3(1000)(-4)(1000 - 4)$$

$$= 1000^3 - 1000^2 \cdot 4 + 1000 \cdot 4^2 - 3(1000)(-4)(1000 - 4)$$

$$= 1000000000 - 160000000 + 4800000 - 12000(1004)$$

$$= 1000000000 - 160000000 + 4800000 - 120480000 = 1012048064.$$

(b)  $(599)^3 = (600 - 1)^3$

$$\because (a - b)^3 = a^3 - b^3 - 3ab(a - b)$$

$$(600 - 1)^3 = (600)^3 - (1)^3 - 3(600 - 1)(600 - 1)$$

$$= 216000000 - 1 - 1800(599)$$

$$= 216000000 - 1 - 1078200$$

$$= 214921799.$$

(c)  $(9.8)^3 = (10 - 0.2)^3$

$$(10)^3 - (0.2)^3 - 3(10 - 0.2)(10 - 0.2)$$

$$= 1000 - 0.008 - 6(10 - 0.2)$$

$$= 1000 - 0.008 - 60 - 1.2$$

$$= 941.192.$$

(d)  $(8.01)^3 = (8.01)^3 - (8 - 0.01)^3$

$$\because (a - b)^3 = a^3 - b^3 - 3ab(a - b)$$

$$(8 - 0.01)^3 = (8)^3 - (0.01)^3 - 3(8 - 0.01)(8 - 0.01)$$

$$= 512 - 0.000001 - 0.24(8.01)$$

$$= 512 - 0.000001 - 1.9224$$

$$= 513.922401.$$

3.  $x - y = 5$

On cubing both sides.

$$(x - y)^3 = (5)^3$$

$$x^3 - y^3 - 3xy(x - y) = 125$$

$$x^3 - y^3 - 3(6)(5) = 125 \quad (\because x - y = 5 \text{ and } xy = 6)$$

$$x^3 - y^3 = 125 - 90 - 35.$$

4.  $x - y = 12$

On cubing both sides

$$(x - y)^3 = (12)^3$$

$$x^3 - y^3 - 3xy(x - y) = 1728$$

$$x^3 - y^3 - 3 \cdot 27(12) = 1728 \quad (\because xy = 27 \text{ and } x - y = 12)$$

$$x^3 - y^3 = 1728 - 972 - x^3 - y^3 = 756.$$

5.  $x - y = 4$

On cubing both sides.

$$(x - y)^3 = (4)^3$$

$$x^3 - y^3 - 3(xy)(x - y) = 64$$

$$x^3 - y^3 - 3(21)(4) = 64 \quad (\because xy = 21, x - y = 4)$$

$$x^3 - y^3 = 64 - 252 + 64$$

$$x^3 - y^3 = 252 - 64$$

$$x^3 - y^3 = 188.$$

6.  $3x - 2y = 11$

On cubing with sides.

$$\begin{array}{rcl} (3x - 2y)^3 & (11)^3 \\ (3x)^3 - (2y)^3 & 3(3x - 2y)(3x - 2y) & 1331 \\ 27x^3 - 8y^3 & 18xy(3x - 2y) & 1331 \\ 27x^3 - 8y^3 & 18 & 12 & 11 & 1331 \\ & 27x^3 - 8y^3 & 1331 & 2376 \\ & 27x^3 - 8y^3 & 3707. & \end{array} \quad (\because xy = 12 \text{ and } 3x - 2y = 11)$$

7.  $x - \frac{1}{x} = 7$

On cubing both sides.

$$\begin{array}{rcl} x - \frac{1}{x} & (7)^3 \\ x^3 - \frac{1}{x^3} & 3 & x - \frac{1}{x} & x - \frac{1}{x} & 343 \\ x^3 - \frac{1}{x^3} & 3 & x - \frac{1}{x} & x - \frac{1}{x} & 343 \\ x^3 - \frac{1}{x^3} & (3 - 7) & 343 & & \end{array} \quad \therefore x - \frac{1}{x} = 7$$
$$\begin{array}{rcl} x^3 - \frac{1}{x^3} & 343 & 21 \\ x^3 - \frac{1}{x^3} & 322. & \end{array}$$

8.  $x - \frac{1}{x} = 5$

On cubing both sides.

$$\begin{array}{rcl} x - \frac{1}{x} & (5)^3 \\ \therefore (a - b)^3 & a^3 - b^3 & 3ab(a - b) \\ x^3 - \frac{1}{x^3} & 3 & x - \frac{1}{x} & x - \frac{1}{x} & 125 \\ x^3 - \frac{1}{x^3} & 3 & x - \frac{1}{x} & x - \frac{1}{x} & 125 \\ x^3 - \frac{1}{x^3} & 3 & 5 & 125 & \\ x^3 - \frac{1}{x^3} & 125 & 15 & & \\ x^3 - \frac{1}{x^3} & 140. & & & \end{array}$$

$$9. \quad x^2 - \frac{1}{x^2} = 7$$

On putting

$$\begin{array}{rcl} x - \frac{1}{x} & = & x^2 - \frac{1}{x^2} = 2 \\ x - \frac{1}{x} & = & 7 - 2 \\ x - \frac{1}{x} & = & 9 \\ x - \frac{1}{x} & = & 3 \end{array} \quad \therefore x - \frac{1}{x} = 3$$

Now, on cubing the both sides

$$\begin{array}{rcl} x - \frac{1}{x} & = & (3)^3 \\ x^3 - \frac{1}{x^3} & = & 3x - \frac{1}{x} = x - \frac{1}{x} = 27 \\ x^3 - \frac{1}{x^3} & = & 3(3) = 27 & \therefore x - \frac{1}{x} = 3 \\ x^3 - \frac{1}{x^3} & = & 9 = 27 \\ x^3 - \frac{1}{x^3} & = & 27 - 9 \\ x^3 - \frac{1}{x^3} & = & 18. \end{array}$$

$$10. \quad x^2 - \frac{1}{x^2} = 27$$

$$\text{On putting } x - \frac{1}{x} = x^2 - \frac{1}{x^2} = 2$$

$$\begin{array}{rcl} \therefore x^2 - \frac{1}{x^2} & = & 27 \\ x - \frac{1}{x} & = & 27 - 2 \\ x - \frac{1}{x} & = & 25 \\ x - \frac{1}{x} & = & 5. \end{array}$$

Now, on cubing both sides

$$\begin{array}{rcl} x - \frac{1}{x} & = & (5)^3 \\ x^3 - \frac{1}{x^3} & = & 3x - \frac{1}{x} = x - \frac{1}{x} = 125 \\ x^3 - \frac{1}{x^3} & = & 3(5) = 125 & \therefore x - \frac{1}{x} = 5 \\ x^3 - \frac{1}{x^3} & = & 125 - 15 \end{array}$$

$$\begin{aligned}
 & x^3 - \frac{1}{x^3} = 140 \\
 11. \quad (a) \quad & (a - 3b)^3 - (a + 3b)^3 \\
 & [a^3 - (3b)^3 - 3(a - 3b)(a + 3b)] - [a^3 + (3b)^3 + 3(a - 3b)(a + 3b)] \\
 & [a^3 - 27b^3 - 9ab(a + 3b)] - [a^3 + 27b^3 + 9ab(a + 3b)] \\
 & a^3 - 27b^3 - 9a^2b - 27ab^2 - a^3 - 27b^3 - 9a^2b - 27ab^2 \\
 & 2a^3 - 54ab^2. \\
 (b) \quad & \frac{1}{3}a^{\frac{2}{3}}b^{\frac{3}{3}} - \frac{1}{3}a^{\frac{2}{3}}b^{\frac{3}{3}} \\
 & \frac{1}{3}a^{\frac{3}{3}} - \frac{2}{3}b^{\frac{3}{3}} = 3 \cdot \frac{a}{3} \cdot \frac{2b}{3} - \frac{a}{3} \cdot \frac{2b}{3} \\
 & \frac{1}{3}a^{\frac{3}{3}} - \frac{2}{3}b^{\frac{3}{3}} = 3 \cdot \frac{a}{3} \cdot \frac{2b}{3} - \frac{a}{3} \cdot \frac{2b}{3} \\
 & \frac{1}{27}a^3 - \frac{8b^3}{27} - \frac{2ab}{3} \cdot \frac{a}{3} \cdot \frac{2b}{3} = \frac{a^3}{27} - \frac{8b^3}{27} - \frac{2}{3}ab \cdot \frac{a}{3} \cdot \frac{2b}{3} \\
 & \frac{a^3}{27} - \frac{8b^3}{27} - \frac{2a^2b}{9} - \frac{4ab^2}{9} = \frac{a^3}{27} - \frac{8b^3}{27} - \frac{2a^2b}{9} - \frac{4ab^2}{9} \\
 & \frac{a^3}{27} - \frac{8b^3}{27} - \frac{2a^2b}{9} - \frac{4ab^2}{9} = \frac{a^3}{27} - \frac{8b^3}{27} - \frac{2a^2b}{9} - \frac{4ab^2}{9} \\
 & 2 \cdot \frac{8b^3}{27} - 2 \cdot \frac{2a^2b}{9} = \frac{16b^3}{27} - \frac{4a^2b}{9}.
 \end{aligned}$$

### Exercise 7.6

1. (a)  $12p^2q^2 \cdot 2 \cdot 2 \cdot 3 \cdot p \cdot p \cdot q$   
 (b)  $16xy^2z^2 \cdot 2 \cdot 2 \cdot 2 \cdot x \cdot y \cdot y \cdot z$   
 (c)  $20a^2b^2c^2 \cdot 2 \cdot 2 \cdot 5 \cdot a \cdot a \cdot b \cdot b \cdot c \cdot c$   
 (d)  $2lm^2np^2 \cdot 3 \cdot 7 \cdot m \cdot m \cdot n \cdot p \cdot p$
2. (a) Common factors of  $2xy$  and  $12x^2y$      $2xy$   
 (b) Common factors of  $3m^2$  and  $15m^4$      $5m^2$   
 (c) Common factors of  $3ax^2y$  and  $18axy$      $3axy$   
 (d) Common factors of  $25p^2q^4$  and  $15pq^2$      $5pq^2$
3. (a)  $3x^2(2 - 5x - 7x^2)$   
 Common factors of  $6x^2 - 15x^3 - 21x^4 - 3x^2$   
 (b)  $9x^2y^2(y - 3x - 4)$   
 Common factors of  $9x^2y^3 - 18x^3y^2 - 36x^2y^2 - 9x^2y^2$   
 (c)  $5a(a^2bc - 3b^3 - 5a^2)$   
 Common factors of  $5a^3bc - 15ab^3 - 25a^3 - 5a$

(d)  $8(p^3 - 29^3 - 4r^3)$

Common factors of  $8p^3 - 16q^3 - 32r^3$

4. (a)  $(x - 3)x - (x - 3)y - (x - y)(x - 3)$

(b)  $3a(x - 4y) - 2b(x - 4y) - (3a - 2b)(x - 4y)$

(c)  $4(a - 2b) - 8(a - 2b)^2 - 4(a - 2b)[1 - 2(a - 2b)] - 4(a - 2b)[1 - 2a + 4b]$

(d)  $5(m - n)^2 - 6(m - n) - (m - n)[5(m - n) - 6] - (m - n)[5m - 5n - 6]$

5. (a)  $abc - ab - c - 1 - ab(c - 1) - 1(c - 1) - (ab - 1)(c - 1)$

(b)  $pq^2 - pr^2 - pq - r^2 - p^2q - pq - pr^2 - r^2$

$\quad \quad \quad pq(p - 1) - r^2(p - 1) - (p - r^2)(p - 1)$

(c)  $4x^2 - 2y^2 - x^2y^2 - 8 - 4x^2 - x^2y^2 - 2y^2 - 8$

$\quad \quad \quad x^2(4 - y^2) - 2(y^2 - 4)$

$\quad \quad \quad (x^2 - 2)(y^2 - 4)$

(d)  $ax^2 - by^2 - bx^2 - ay^2 - ax^2 - bx^2 - ay^2 - by^2$

$\quad \quad \quad x^2(a - b) - y^2(a - b)$

$\quad \quad \quad (x^2 - y^2)(a - b)$

6. (a)  $x^2 - 16 - (x - 4)^2 - (4)^2$  [Using  $x^2 - y^2 = (x - y)(x + y)$ ]

(b)  $4 - 36y^2 - (2)^2 - (6y)^2$

$\quad \quad \quad (2 - 6y)(2 + 6y)$  [Using  $x^2 - y^2 = (x - y)(x + y)$ ]

(c)  $a^4b^4 - c^4 - (a^2b^2)^2 - (c^2)^2$  [Using  $a^2 - b^2 = (a - b)(a + b)$ ]

$\quad \quad \quad (a^2b^2 - c^2)(a^2b^2 + c^2)$

$\quad \quad \quad (a^2b^2 - c^2)(a^2b^2 - c^2)(a^2b^2 + c^2)$

(d)  $m^2 - (n - p)^2 - (m)^2 - (n - p)^2$  [Using  $a^2 - b^2 = (a - b)(a + b)$ ]

$\quad \quad \quad (m - n + p)(m - n - p)$

(e)  $8p^3 - 2p - 2p(4p^2 - 1)$  [Using  $a^2 - b^2 = (a - b)(a + b)$ ]

$\quad \quad \quad 2p((2p)^2 - (1)^2)$

$\quad \quad \quad 2p(2p - 1)(2p + 1)$

(f)  $16x^4 - (z - x)^4 - (4x^2)^2$   $[(z - x)^2]^2$

$\quad \quad \quad (4x^2 - (z - x)^2)[4x^2 - (2x)^2]$

$\quad \quad \quad [4x^2 - (z - x)(z - x)][4x^2 - (z - x)(z - x)]$

7. (a)  $y^2 - 18y - 81$

$\quad \quad \quad (y)^2 - 2y - 9 - (9)^2$  [Using  $(a - b)^2 = a^2 - 2ab + b^2$ ]

$\quad \quad \quad (y - 9)^2$

(b)  $x^4 - 22x^2 - 121$

$\quad \quad \quad (x^2)^2 - 2x - 11 - (11)^2$  [Using  $(a - b)^2 = a^2 - 2ab + b^2$ ]

$\quad \quad \quad (x^2 - 11)^2$

(c)  $p^6 - 4p^3 - 4$

$\quad \quad \quad (p^3)^2 - 2 - p^3 - 2 - (2)^2$  [Using  $(a - b)^2 = a^2 - 2ab + b^2$ ]

$$(p^3 - 2)^2$$

(d)  $a^2 - 2ab + b^2 - 16$

$$(a^2 - 2ab + b^2) - (4)^2$$

[Using  $(a - b)^2 = a^2 - 2ab + b^2$ ]

$$(a - b)^2 - (4)^2$$

[Using  $(a^2 - b^2) = (a - b)(a + b)$ ]

$$(a - b - 4)(a + b - 4)$$

(e)  $9z^2 - x^2 - 4y^2 - 4xy$

$$9z^2 - (x^2 - 4y^2 - 4xy)$$

[Using  $(a - b)^2 = a^2 - 2ab + b^2$ ]

$$9z^2 - (x^2 - (2y)^2 - 2x - 2y)$$

$$9z^2 - (x - 2y)^2$$

[Using  $(a^2 - b^2) = (a - b)(a + b)$ ]

$$(3z)^2 - (x - 2y)^2$$

$$(3z - x + 2y)(3z + x - 2y)$$

(f)  $x^8 - y^8$

$$x^4 - y^4$$

$$(x^4)^2 - (y^4)^2 - (x^2)^2 - (y^2)^2$$

[Using  $(a^2 - b^2) = (a - b)(a + b)$ ]

$$(x^4 - y^4)(x^4 + y^4) - (x^2 - y^2)(x^2 + y^2)$$

$$(x^4 - y^4)(x^2 - y^2)(x^2 + y^2) - (x^2 - y^2)$$

$$(x^2 - y^2)(x^4 - y^4 - 1) - (x^2 - y^2)^2$$

$$(x^2 - y^2)(x^2 - y^4 - 1) - (x - y)(x + y)$$

8. (a)  $x^2 - 9x - 20$

(b)  $x^2 - 14x - 13$

$$x^2 - 5x - 4x - 20$$

$$x^2 - 13x - x - 13$$

$$x(x - 5) - 4(x - 5)$$

$$x(x - 13) - 1(x - 13)$$

$$(x - 4)(x - 5)$$

$$(x - 1)(x - 13)$$

(c)  $p^2 - 2p - 15$

(d)  $m^2 - 11mn - 18n^2$

$$p^2 - 2p - 15$$

$$m^2 - 9mn - 2mn - 18n^2$$

$$p(p - 3) - 5(p - 3)$$

$$m(m - 9) - 2n(m - 9n)$$

$$(p - 5)(p - 3)$$

$$(m - 9n)(m - 2n)$$

(e)  $m^2 - 3m - 70$

(f)  $3x^2 - 10x - 8$

$$m^2 - 10m - 7m - 70$$

$$3x^2 - 74x - 4x - 8$$

$$m(m - 10) - 7(m - 10)$$

$$3x(x - 2) - 4(x - 2)$$

$$(3x - 4)(x - 2)$$

$$(3x - 4)(x - 2)$$

$$(m - 7)(m - 10)$$

(g)  $10p^2 - 11p - 3$

(h)  $11a^2 - 54a - 63$

$$10p^2 - 5p - 6p - 3$$

$$11a^2 - 21a - 33a - 63$$

$$5p(2p - 1) - 3(2p - 1)$$

$$a(11a - 2a) - 3(11a - 21)$$

$$5p - 3(2p - 1)$$

$$(a - 3)(11a - 21)$$

(i)  $12y^2 - 28y - 5 - 12y^2 - 2y - 30y - 5 - 2y(6y - 1) - 5(6y - 1)$

$$(2y - 5)(6y - 1)$$

### MCQ's

1. (b)

2. (a)

3. (c)

4. (d)

5. (a)

6. (c)

7. (d)

8. (d)